# COORDINATE GEOMETRY

PALMS

# TEACHER'S REMINDER

Remember to have the following tools ready before starting your lesson:

1. YoTeach chatroom
2. Realtimeboard
3. Badaboom Quiz

(see lesson plan for details)

# COORDINATE GEOMETRY

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# LESSON OBJECTIVES

- ☐ Identify the coordinates of a point on a plane
- ☐ Calculate the distance between two points on a plane
- ☐ Find the mid-point between two points on a plane

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# CLASS ACTIVITY: REALTIMEBOARD

- Choose a boat sticker and place on the coordinate plane.

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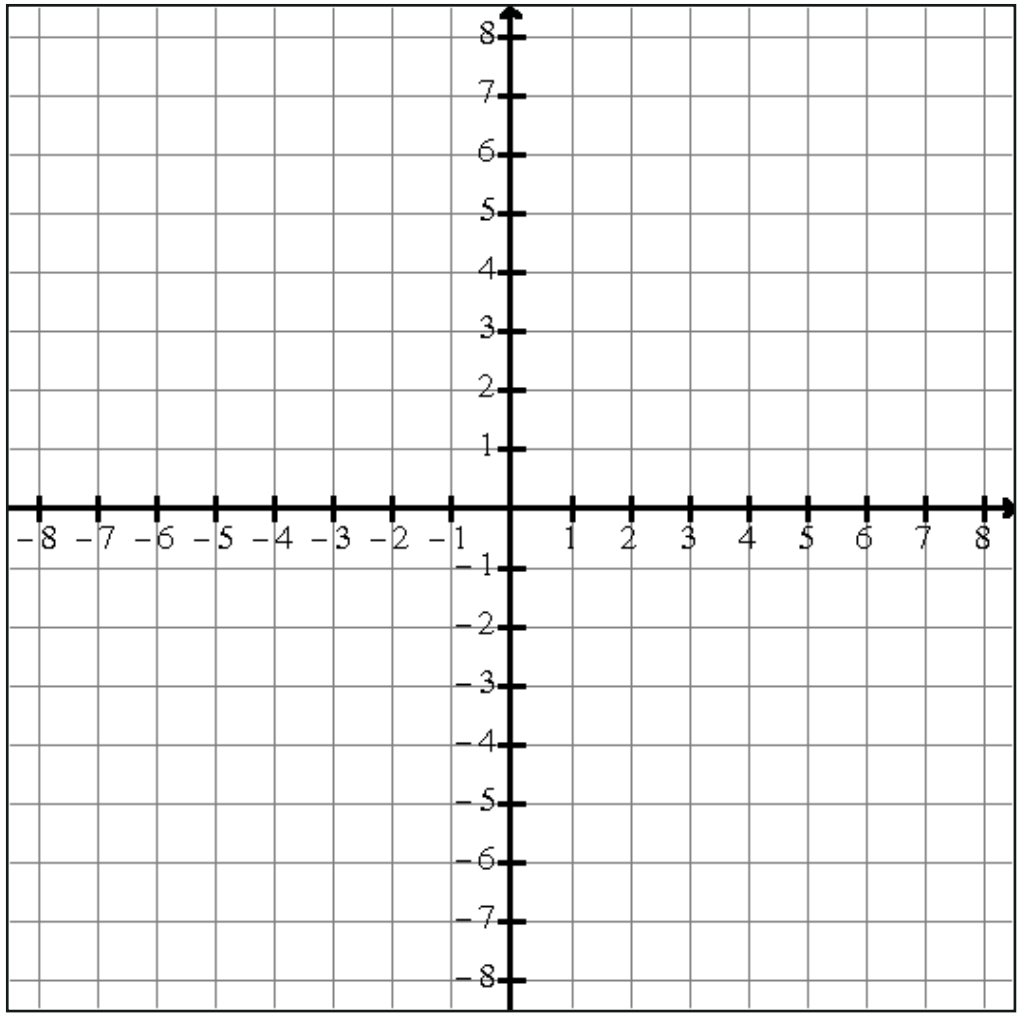
# POINTS AND THEIR COORDINATES




## Cartesian Plane

René Descartes (1596-1650)

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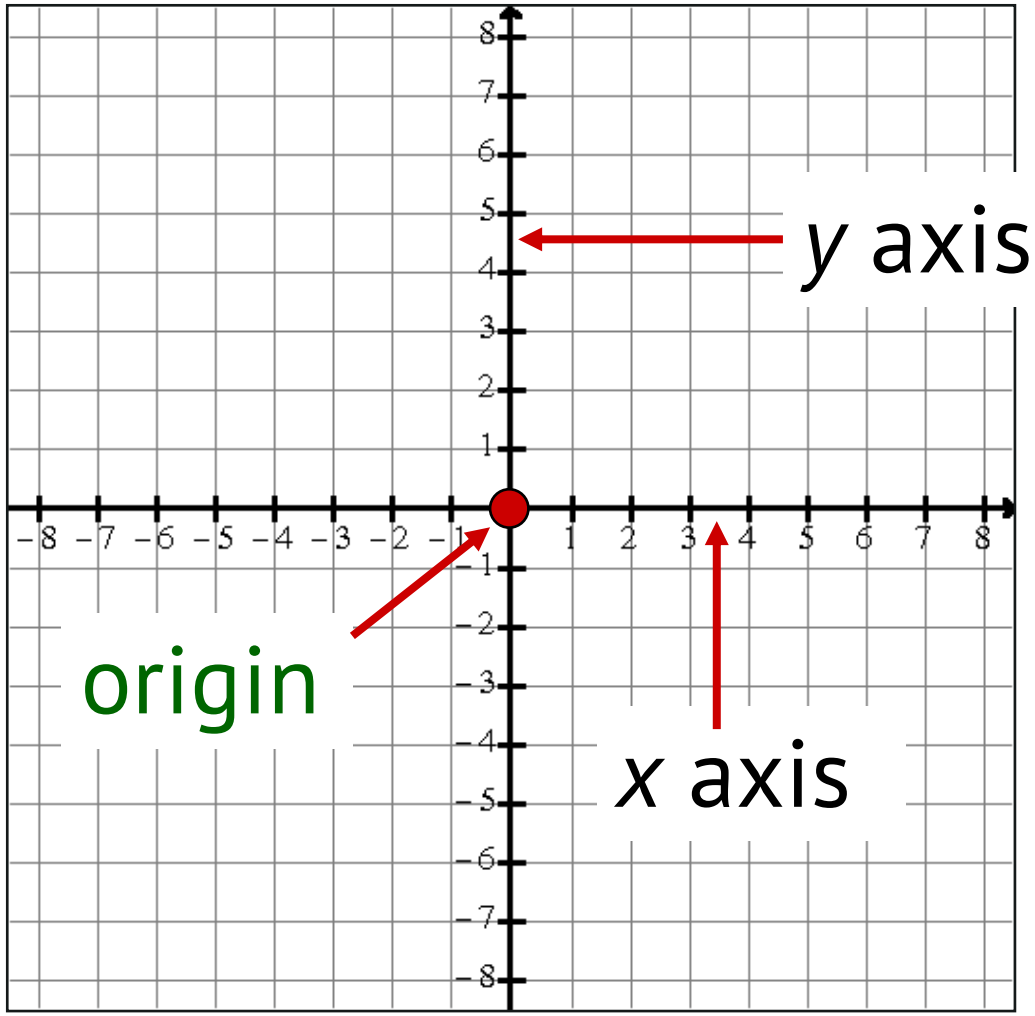
# POINTS AND THEIR COORDINATES




## Cartesian Plane

René Descartes (1596-1650)

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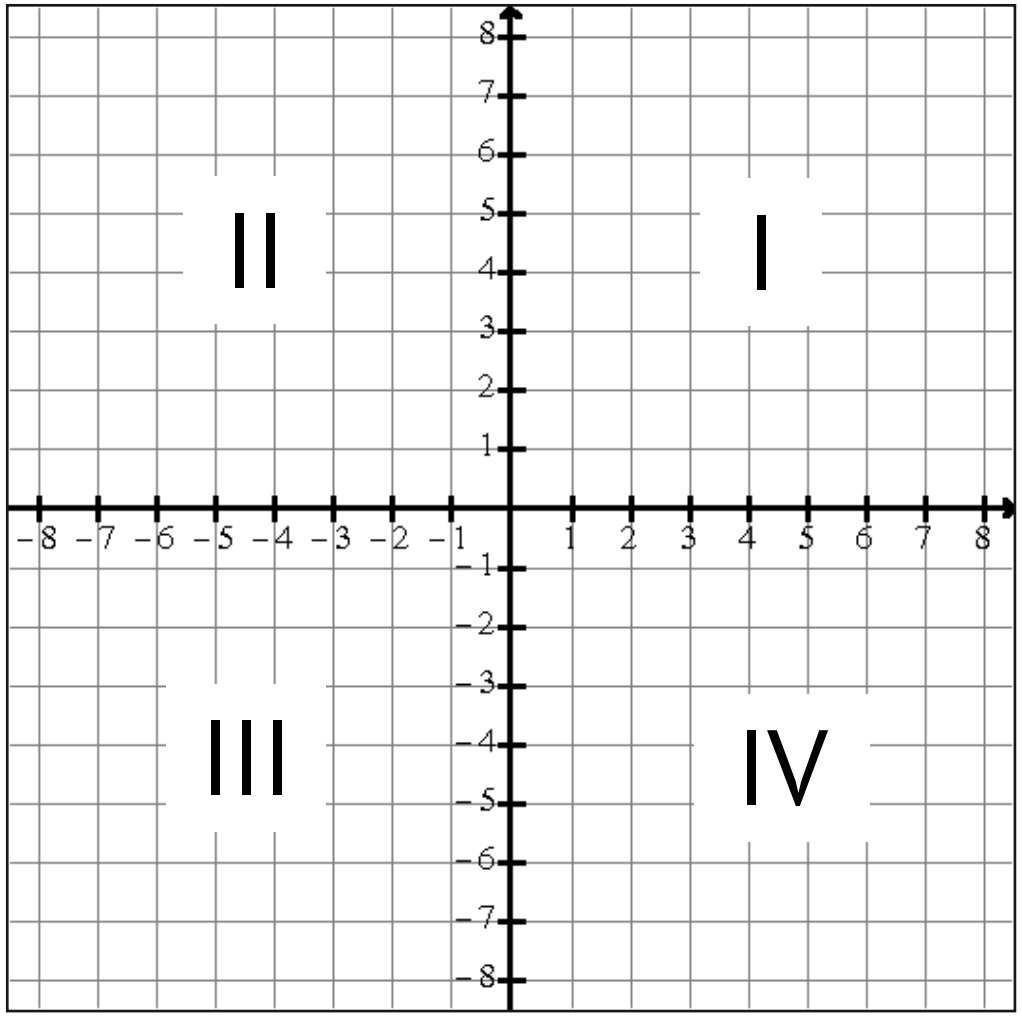
# CARTESIAN PLANE




- This plane is divided into 4 quadrants.

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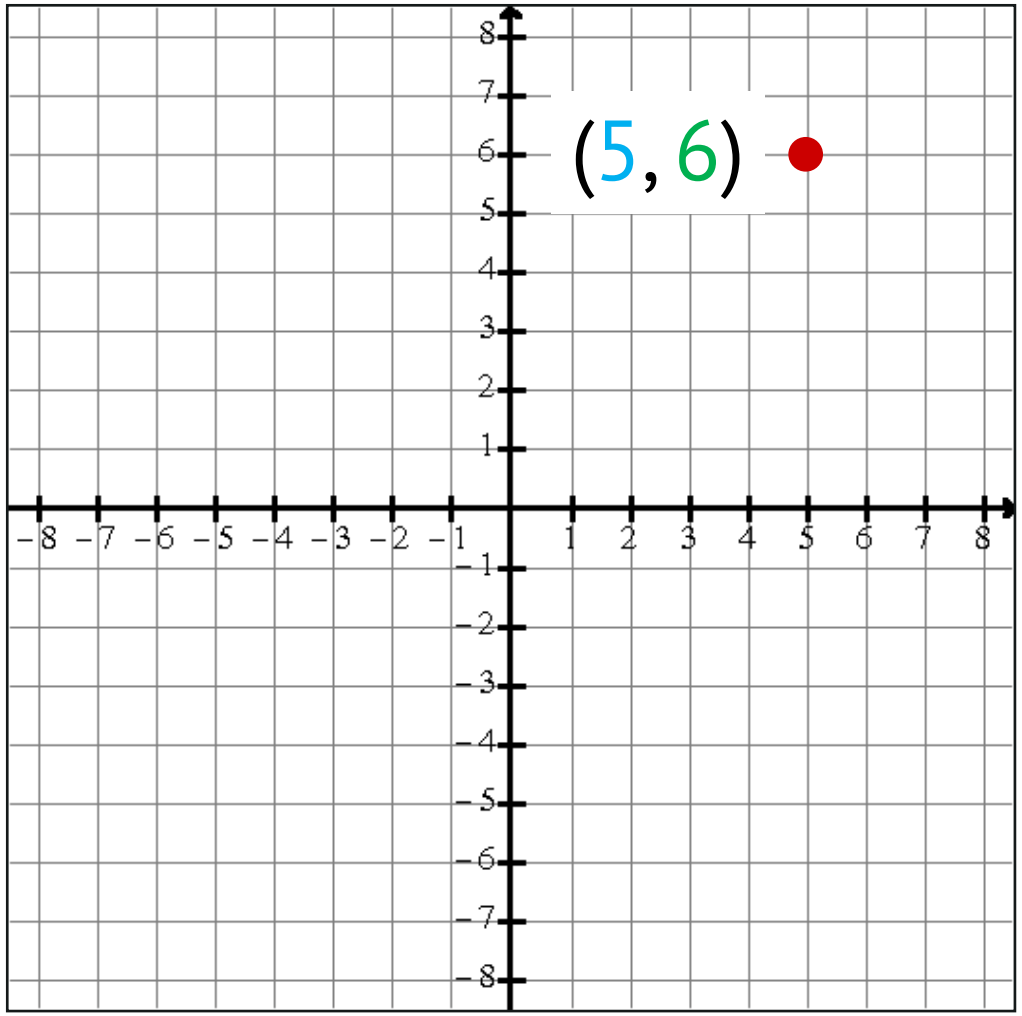
# CARTESIAN PLANE




- Points in this plane come in **ordered pairs**.
- They are plotted according to its x- and y-coordinates.
- Example: ( 5 , 6 )

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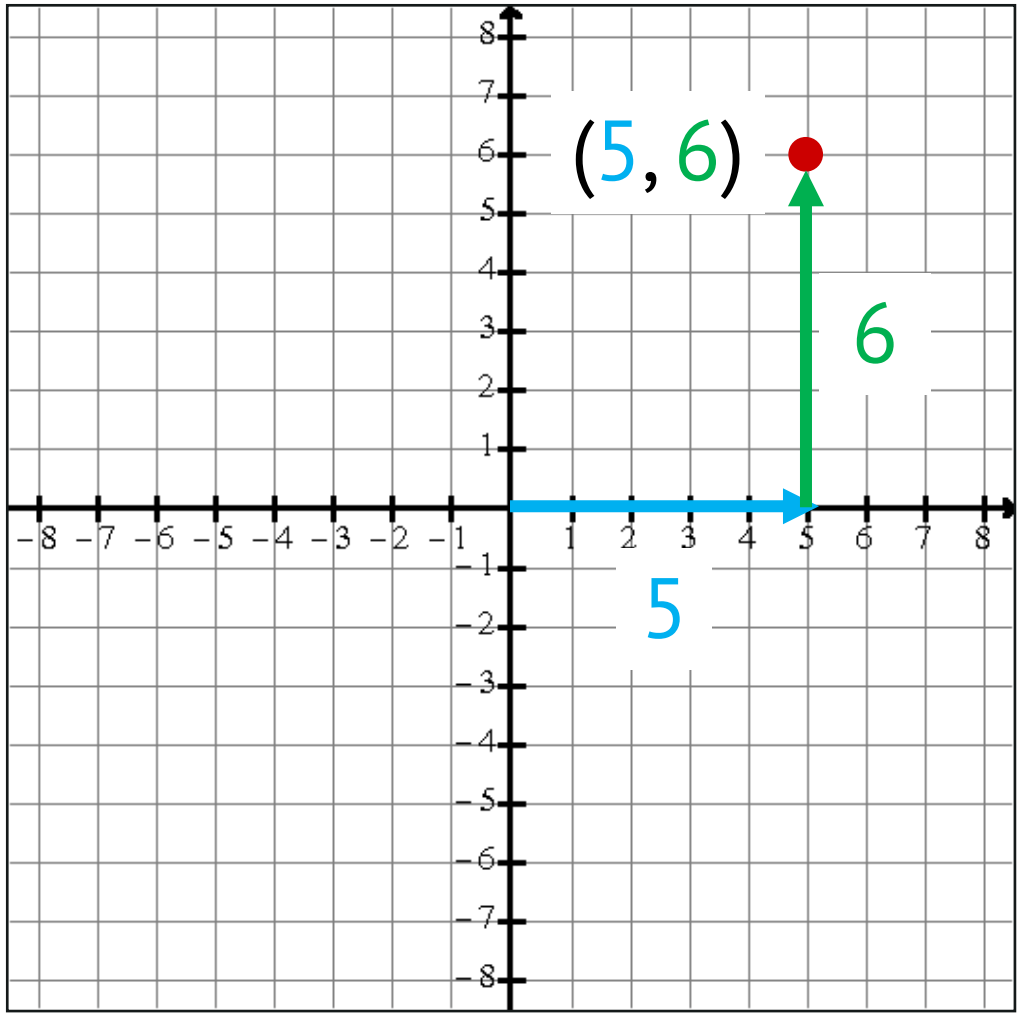
# CARTESIAN PLANE




- Points in this plane come in **ordered pairs**.
- They are plotted according to its x- and y-coordinates.
- Example: ( 5 , 6 )

*x*-coordinate

*y*-coordinate

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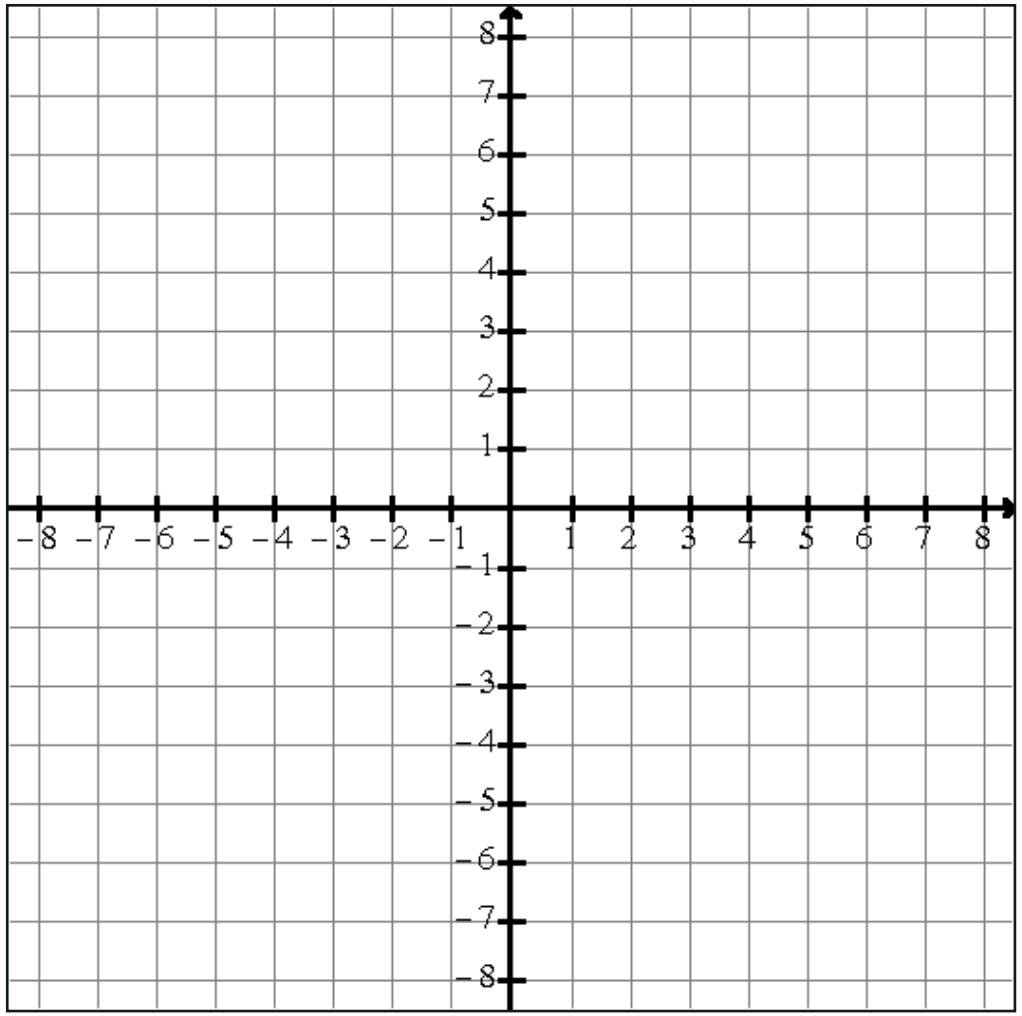
# PLOT THE FOLLOWING POINTS




- $E(5, 0)$
- $F(0, 6)$
- $G(-7, 0)$
- $H(0, -3)$

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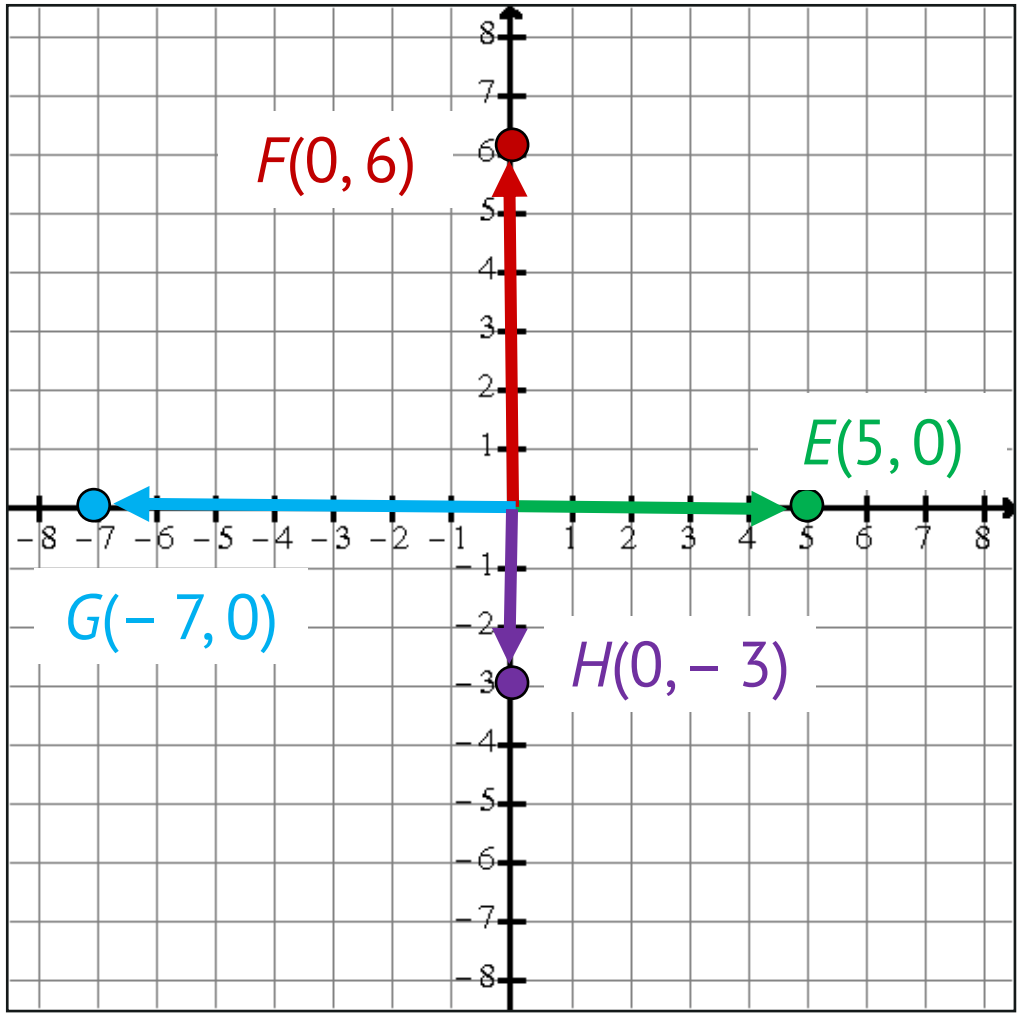


- $E(5, 0)$
- $F(0, 6)$
- $G(-7, 0)$
- $H(0, -3)$

- These points all lie on the axes, not in quadrants. What do you notice about their coordinates?

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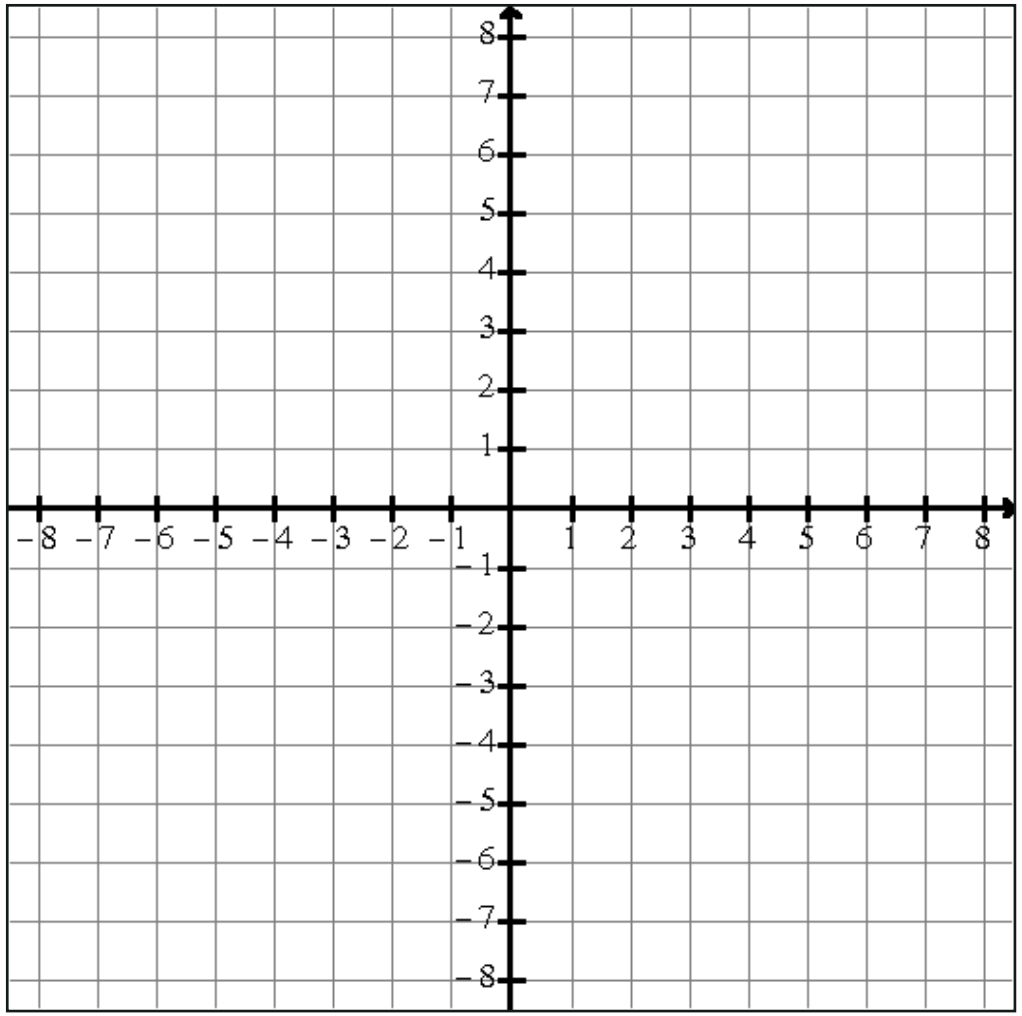
# PLOT THE FOLLOWING POINTS




- *A* (– 4, 6)
- *B* (2, – 3)
- *C* (– 6, – 4)
- *D* (7, 3)

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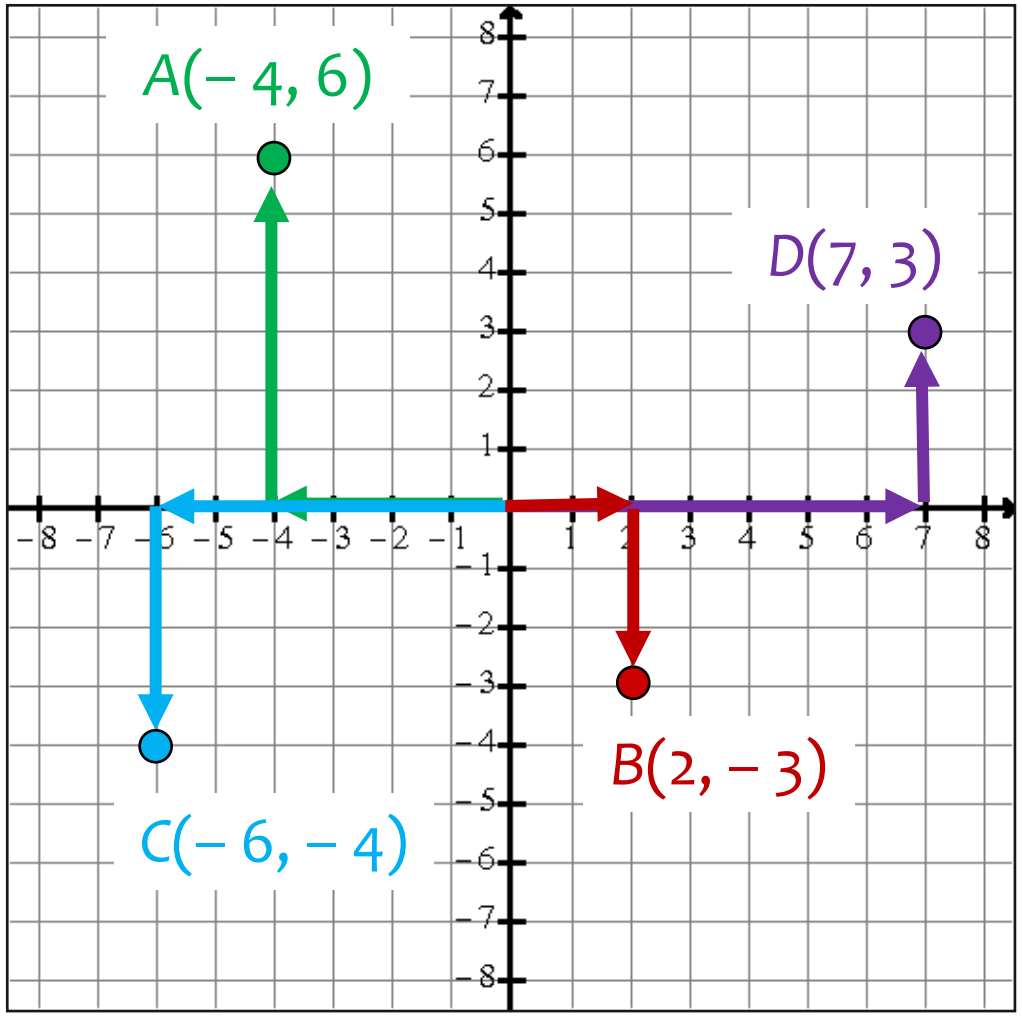
# PLOT THE FOLLOWING POINTS

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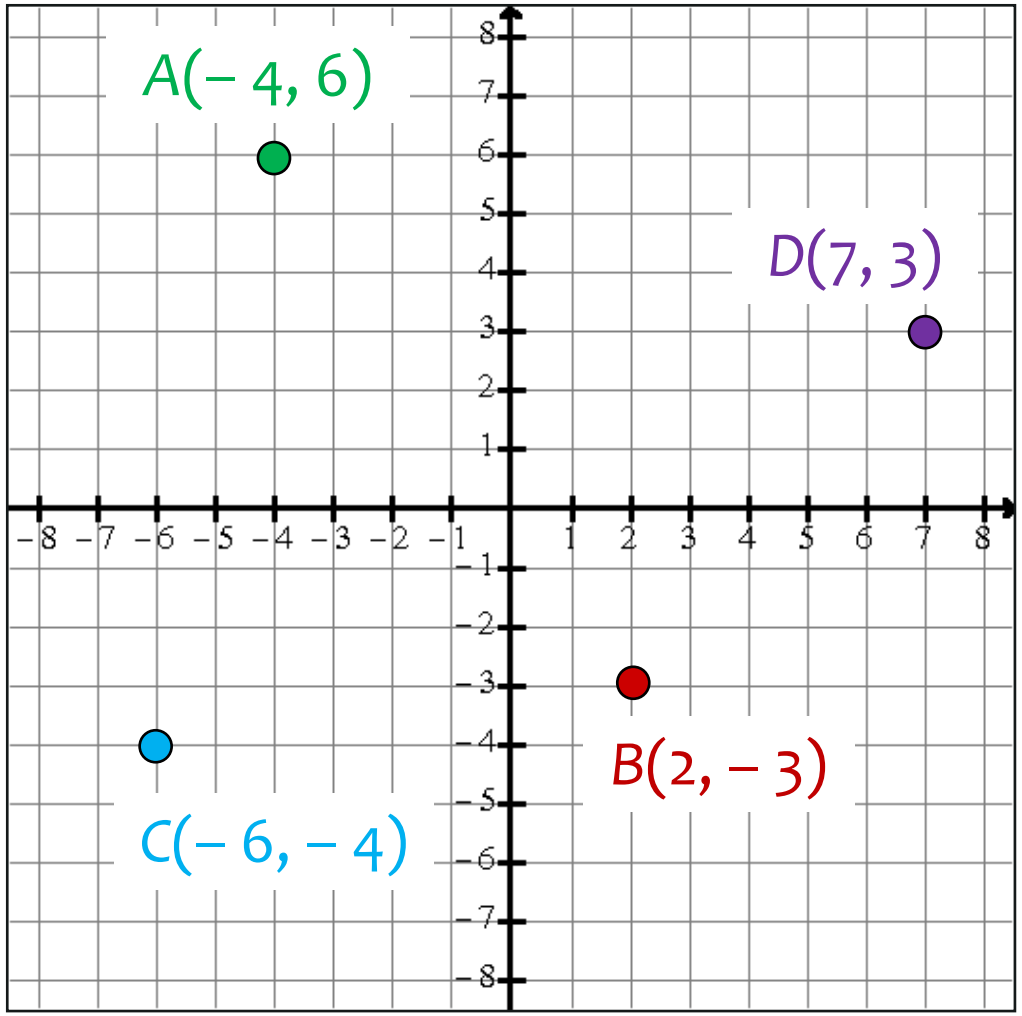


- *A* (– 4, 6)
- *B* (2, – 3)
- *C* (– 6, – 4)
- *D* (7, 3)

- These points all lie in different quadrants. What do you notice about their coordinates?

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# THE DISTANCE FORMULA

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# THE DISTANCE FORMULA

- We know on a 1-dimensional number line, the distance between points $a$ and $b$ is:

$$d(a, b) = |b - a|$$

- What about the distance between 2 points on a 2D plane?

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# THE DISTANCE FORMULA

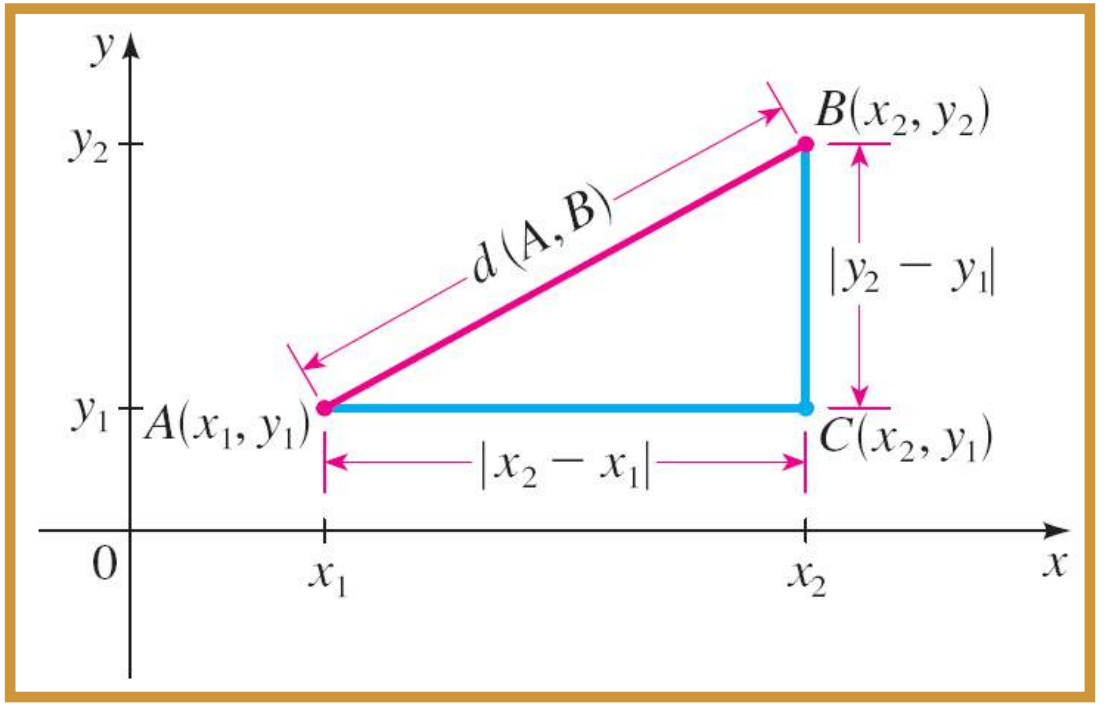
- We 'll need a formula for the distance $d(A, B)$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane.

- Let's try to graph it:

# THE DISTANCE FORMULA

- Starting with our 2 points *A* and *B*, let's form a **right triangle** from these points.

- Let this right angle be *C*.

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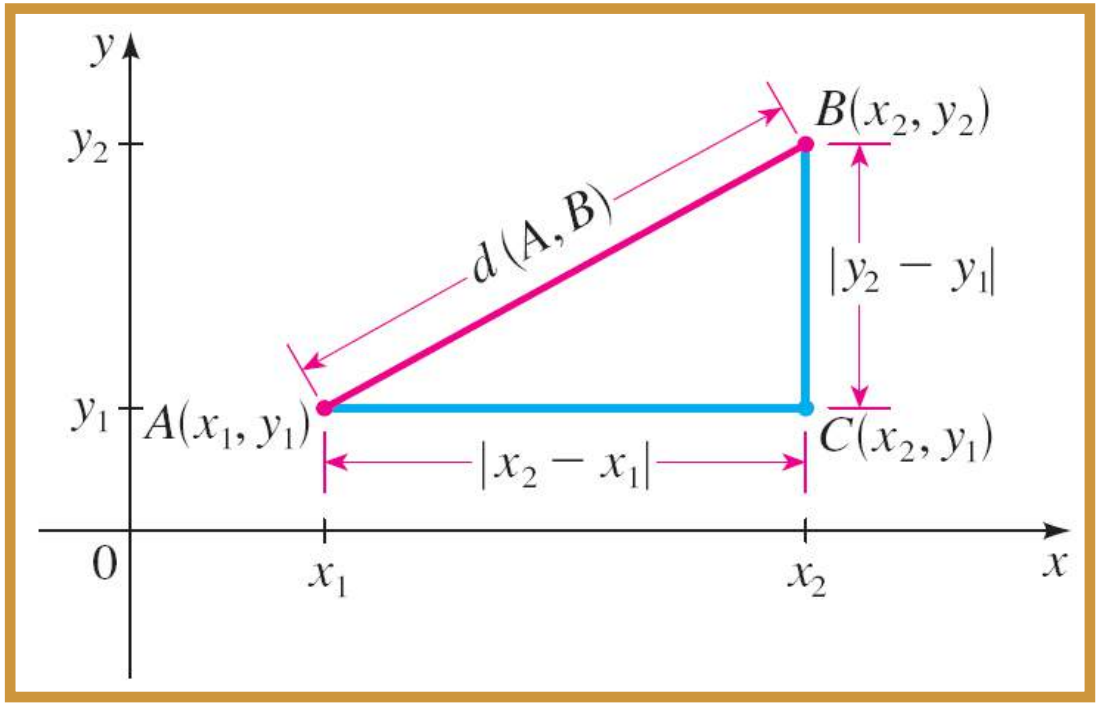
# THE DISTANCE FORMULA

- The distance between $A(x_1, y_1)$ and $C(x_2, y_1)$ on the horizontal line is $|x_2 - x_1|$ .

- The distance between $B(x_2, y_2)$ ) and $C(x_2, y_1)$ on the vertical line is $|y_2 - y_1|$.

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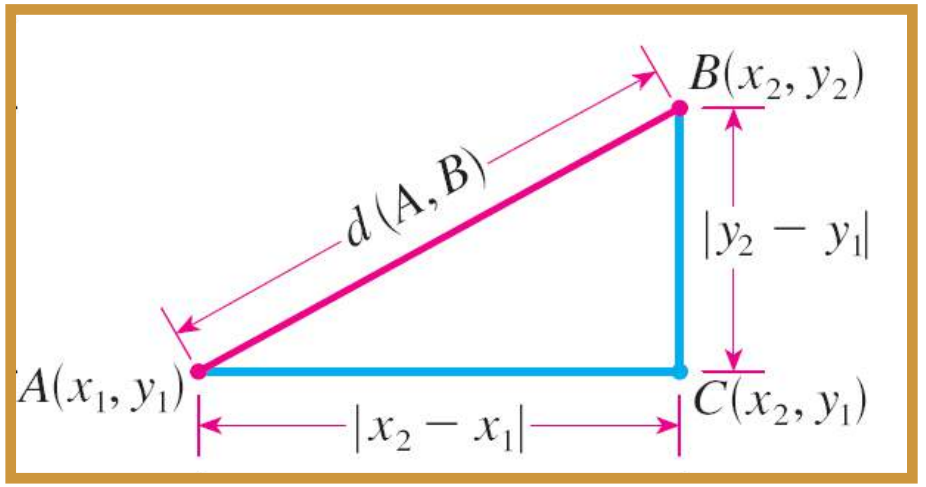
# THE DISTANCE FORMULA

- Since this is a right triangle, we can now use Pythagorean Theorem to calculate $d(A, B)$:




$$d(A, B) = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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# THE DISTANCE FORMULA

- Therefore, the distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is:

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

# EXERCISE 1 – FINDING THE DISTANCE BETWEEN 2 POINTS

- Find the distance between the points $A(2, 5)$ and $B(4, -1)$.

# EXERCISE 1 – FINDING THE DISTANCE BETWEEN 2 POINTS

- Find the distance between the points A(2, 5) and B(4, –1).

- Using the Distance Formula, we have:

$$
\begin{aligned}
d(A,B) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(4-2)^2 + (-1-5)^2} \\
&= \sqrt{2^2 + (-6)^2} \\
&= \sqrt{4 + 36} \\
&= \sqrt{40} \approx 6.32
\end{aligned}
$$

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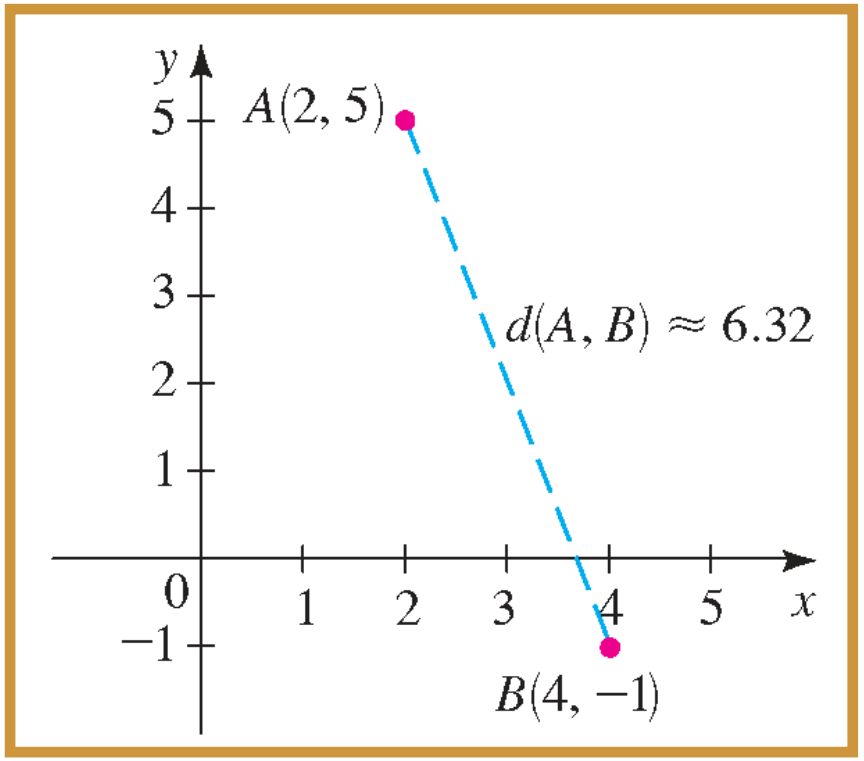
# EXERCISE 1 – FINDING THE DISTANCE BETWEEN 2 POINTS

- We see that the distance between points *A* and *B* is approximately 6.32.

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# EXERCISE 2 – APPLYING THE DISTANCE FORMULA

- Which of the points $P(1, -2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$ ?

# EXERCISE 2 – APPLYING THE DISTANCE FORMULA

- Which of the points *P(1, −2)* or *Q(8, 9)* is closer to the point *A(5, 3)* ?

- By the Distance Formula, we have:

$$d(P,A) = \sqrt{(5-1)^2 + (3-(-2))^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

$$d(Q,A) = \sqrt{(5-8)^2 + (3-9)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45}$$

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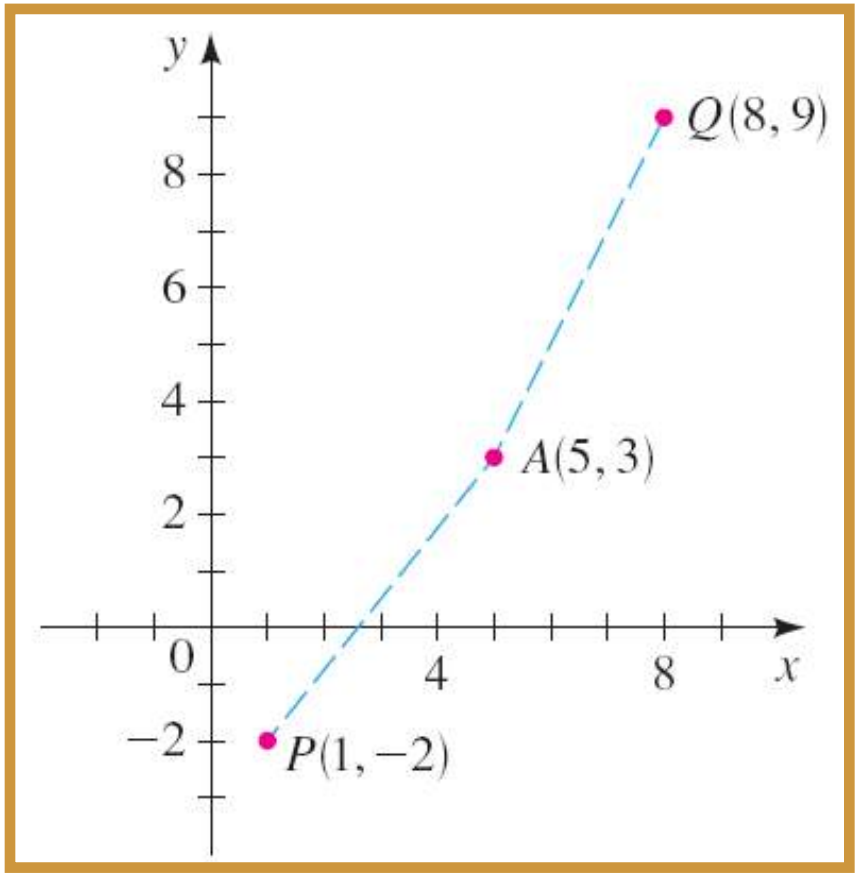
# EXERCISE 2 – APPLYING THE DISTANCE FORMULA

- This shows that $d(P, A) < d(Q, A)$
- So, *P* is closer to *A*.

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# THE MIDPOINT FORMULA

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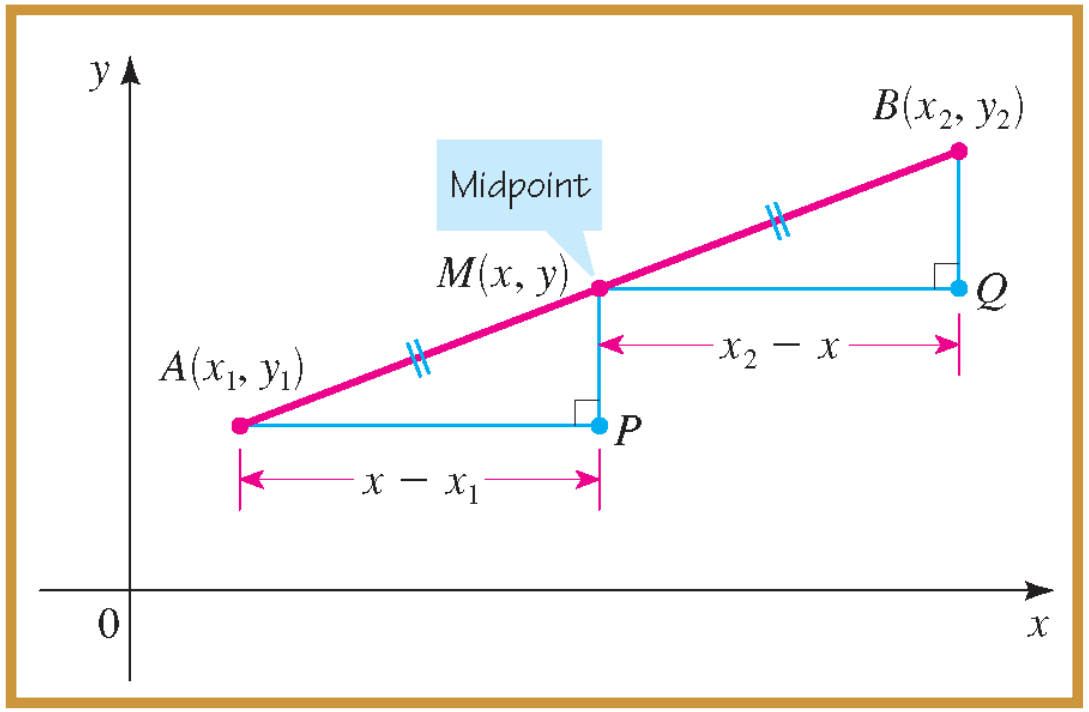
# THE MIDPOINT FORMULA

- This formula will allow us to find the midpoint *M* of a line segment from point $A(x_1, y_1)$ and $B(x_2, y_2)$.

# THE MIDPOINT FORMULA

- We start with our line segment from $A(x_1, y_1)$ and $B(x_2, y_2)$, with midpoint $M(x, y)$ .

- Form 2 right triangles *APM* and *MQB*. Note these are congruent because:
  - $d(A, M) = d(M, B)$
  - The corresponding angles are equal.

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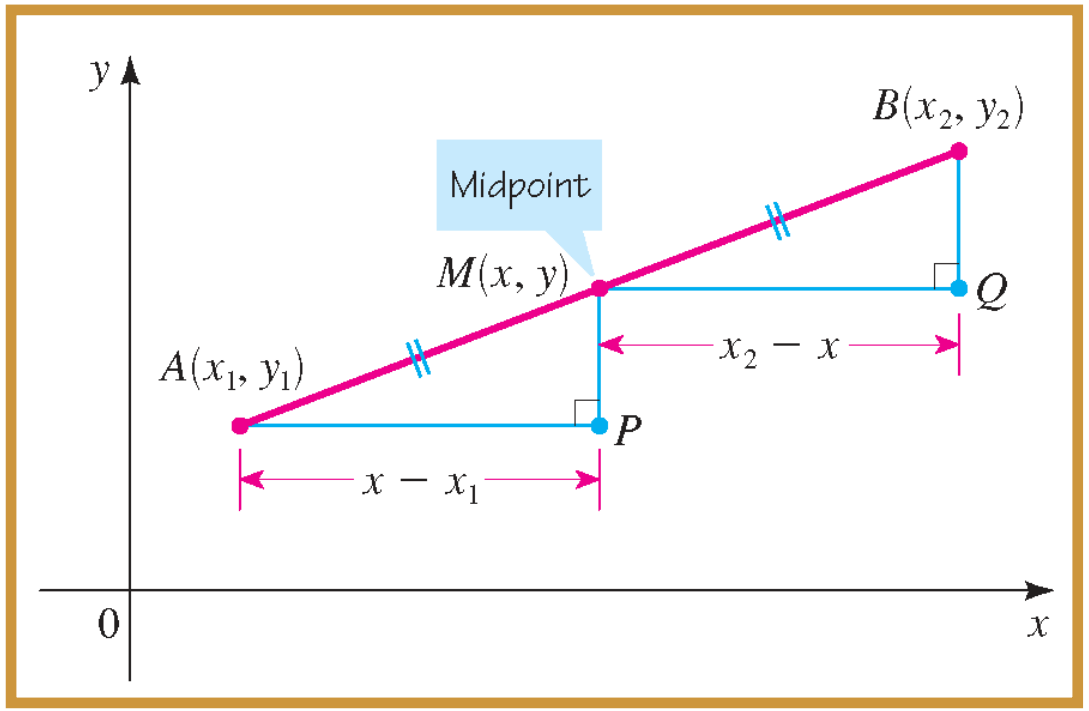
# THE MIDPOINT FORMULA

- Hence,

$$d(A,P) = d(M,Q)$$
$$x - x_1 = x_2 - x$$
$$2x = x_1 + x_2$$
$$x = \frac{x_1 + x_2}{2}$$

Similarly, $y = \frac{y_1+y_2}{2}$.

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# THE MIDPOINT FORMULA

- The midpoint of a line segment from point $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

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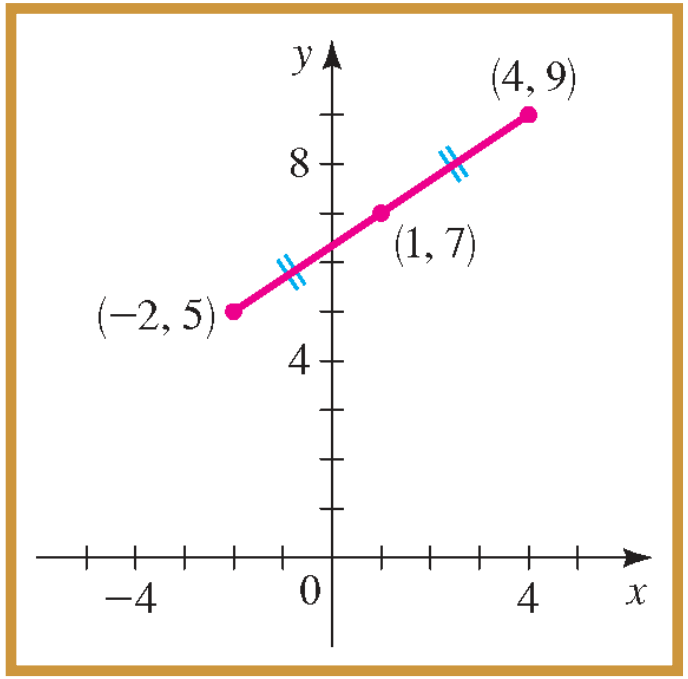
# EXERCISE 3 – FINDING THE MIDPOINT

- Find the midpoint of the line segment that joins the points $(-2, 5)$ and $(4, 9)$.

# EXERCISE 3 – FINDING THE MIDPOINT

- Find the midpoint of the line segment that joins the points $(-2, 5)$ and $(4, 9)$.

$$\left(\frac{-2+4}{2}, \frac{5+9}{2}\right) = (1, 7)$$

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# CLASS ACTIVITY: BADABOOM & REALTIMEBOARD

- Coordinate Geometry Quiz
- Attack a coordinate if you get a question right!


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# CHALLENGE: REGIONS ON THE COORDINATE PLANE

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# EXERCISE 4 – GRAPHING REGIONS IN THE COORDINATE PLANE

- Describe and sketch the regions given by each set:

(a) $\{(x, y) \mid x \geqslant 0\}$

(b) $\{(x, y) \mid y = 1\}$

(c) $\{(x, y) \mid |y| < 1\}$

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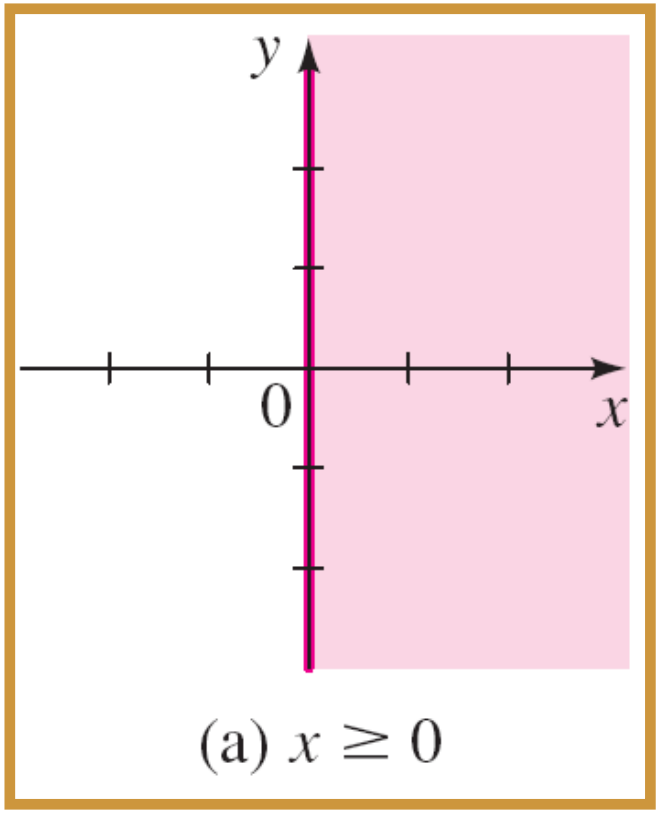
# ANSWER 4A

$\{(x, y) \mid x \geqslant 0\}$

- The points whose x-coordinates are 0 or positive lie on the y-axis or to the right of it.

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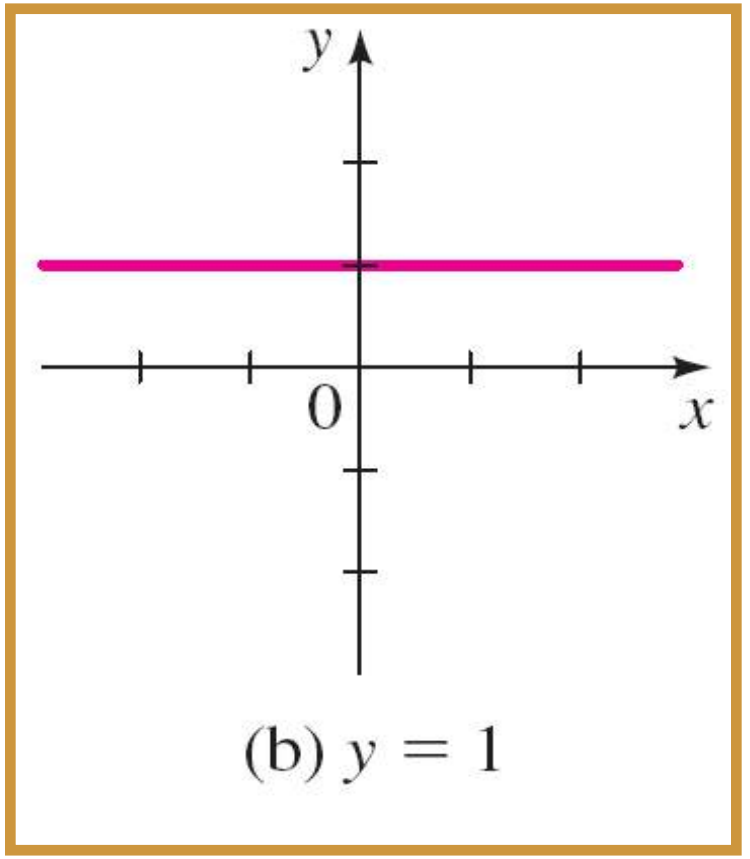
# ANSWER 4B

$\{(x, y) \mid y = 1\}$

- The set of all points with a y-coordinate of 1 is a horizontal line one unit above the x-axis.



# ANSWER 4C

$\{(x, y) \mid |y| < 1\}$

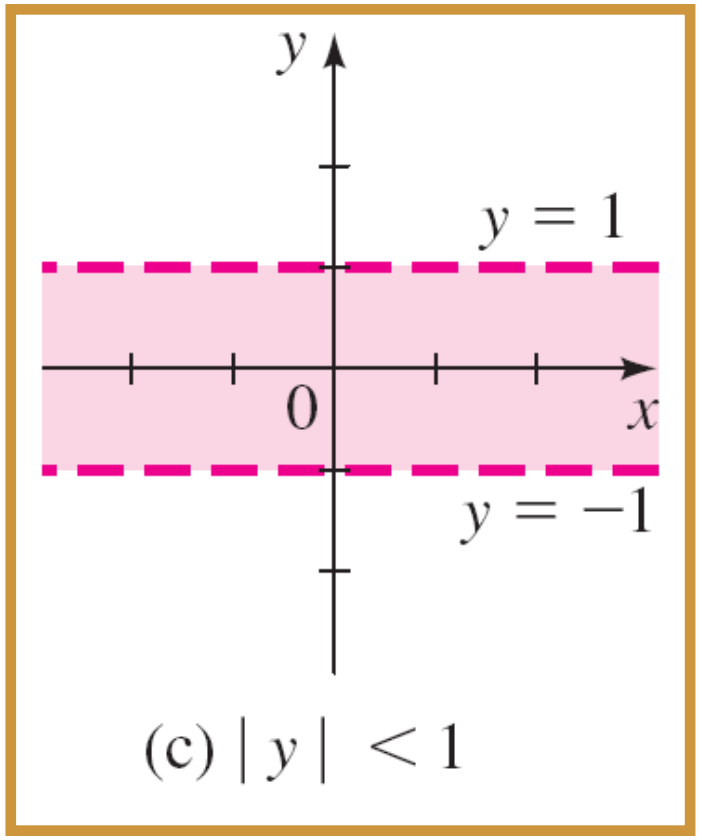
- Recall that: $|y| < 1$ if and only if $-1 < y < 1$

- So, the given region consists of those points in the plane whose $y$-coordinates lie between $-1$ and 1.

# ANSWER 4C

$\{(x, y) \mid |y| < 1\}$

- Thus, the region consists of all points that lie between (but not on) the horizontal lines $y = 1$ and $y = -1$.
- We use dotted lines to indicate that the points on these lines are not included in the set.

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# CLASS ACTIVITY: YOTEACH!

- Reflection: What have you learnt today?


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# LESSON OBJECTIVES

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- ☑ Find the mid-point between two points on a plane

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