QUADRATIC EQUATION II

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TEACHER'S REMINDER

Remember to have the following tools ready before starting your lesson:

1. YoTeach chatroom
2. Realtimeboard
3. Badaboom Quiz

(see lesson plan for details)

QUADRATIC EQUATION II

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LESSON OBJECTIVES

- ☐ Understand the basic concepts of quadratic equations
- ☐ Use completing the square to solve quadratic equations
- ☐ Use the quadratic formula to solve quadratic equations

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CLASS ACTIVITY: BADABOOM

- Quadratic Equations Quiz

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COMPLETING THE SQUARE

- Write the equation in the form: $\boldsymbol{x^2 + bx = c}$
- Add $\left(\frac{\boldsymbol{b}}{\boldsymbol{2}}\right)^{\boldsymbol{2}}$ to each side: $x^2 + bx + \left(\frac{\boldsymbol{b}}{\boldsymbol{2}}\right)^{\boldsymbol{2}} = c + \left(\frac{\boldsymbol{b}}{\boldsymbol{2}}\right)^{\boldsymbol{2}}$
- Factor the LHS: $\left(\boldsymbol{x} + \frac{\boldsymbol{b}}{\boldsymbol{2}}\right)^{\boldsymbol{2}} = c + \left(\frac{b}{2}\right)^2$
- Take the square root of both sides.
- Solve for x.

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SOLVE: $w^2 + 6w + 4 = 0$

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1. Rewrite the equation in the form $x^2 + bx = c$. Here, we move the constant to the right.

$$w^2 + 6w = -4$$

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SOLVE: $w^2 + 6w + 4 = 0$

1. Rewrite the equation in the form $x^2 + bx = c$. Here, we move the constant to the right.

$$w^2 + 6w = -4$$

2. Add $\left(\frac{b}{2}\right)^2$ to each side and factor.

Firstly, let's calculate this: $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 3^2 = 9$

Now let's add this to both sides:

$$w^2 + 6w + 9 = -4 + 9$$

$$(w + 3)^2 = 5$$

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SOLVE: $w^2 + 6w + 4 = 0$

3. Take the square roots of both sides:

$$(w + 3)^2 = 5$$

$$w + 3 = \pm\sqrt{5}$$

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SOLVE: $w^2 + 6w + 4 = 0$

3. Take the square roots of both sides:

$$(w + 3)^2 = 5$$

$$w + 3 = \pm\sqrt{5}$$

4. Solve for *w*:

$$w = -3 \pm \sqrt{5}$$

Solution: $w = -3 + \sqrt{5}$ or $w = -3 - \sqrt{5}$

SOLVE: $2r^2 = 3 - 5r$

1. Rewrite the equation in the form $x^2 + bx = c$.

$$\mathbf{2r^2 + 5r = 3}$$

$$\mathbf{\Rightarrow r^2 + \frac{5}{2}r = \frac{3}{2}}$$

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SOLVE: $2r^2 = 3 - 5r$

1. Rewrite the equation in the form $x^2 + bx = c$.

$$2r^2 + 5r = 3$$

$$\Rightarrow r^2 + \frac{5}{2}r = \frac{3}{2}$$

2. Add $\left(\frac{b}{2}\right)^2$ to each side and factor.

Firstly, $\left(\frac{b}{2}\right)^2 = \left(\frac{1}{2} \cdot \frac{5}{2}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

SOLVE: $2r^2 = 3 - 5r$

Now let's add it to both sides:

$$r^2 + \frac{5}{2}r + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$$

$$\left(r + \frac{5}{4}\right)^2 = \frac{49}{16}$$

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SOLVE: $2r^2 = 3 - 5r$

Now let's add it to both sides:

$$r^2 + \frac{5}{2}r + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$$

$$\left(r + \frac{5}{4}\right)^2 = \frac{49}{16}$$

3. Take the square roots of both sides:

$$\left(r + \frac{5}{4}\right) = \pm\sqrt{\frac{49}{16}} = \pm\frac{7}{4}$$

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SOLVE: $2r^2 = 3 - 5r$

4. Solve for *r*.

$$r = -\frac{5}{4} \pm \frac{7}{4}$$

$$r = -\frac{5}{4} + \frac{7}{4}$$

$$r = \frac{2}{4}$$

$$= \frac{1}{2}$$

$$r = -\frac{5}{4} - \frac{7}{4}$$

$$r = -\frac{12}{4}$$

$$= -3$$

Solution: $r = {}^1/_2$ or $r = -3$

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SOLVE: $3p - 5 = (p - 1)(p - 2)$

1. Rewrite the equation in the form $x^2 + bx = c$. In this case, we need to expand and collect like terms:

$$3p - 5 = p^2 - 2p - p + 2$$
$$3p - 5 = p^2 - 3p + 2$$
$$p^2 - 6p = -7$$

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SOLVE: $3p - 5 = (p - 1)(p - 2)$

1. Rewrite the equation in the form $x^2 + bx = c$. In this case, we need to expand and collect like terms:

$$3p - 5 = p^2 - 2p - p + 2$$
$$3p - 5 = p^2 - 3p + 2$$
$$p^2 - 6p = -7$$

2. Add $\left(\frac{b}{2}\right)^2$ to each side and factor.

Firstly, $\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

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SOLVE: $3p - 5 = (p - 1)(p - 2)$

Now let's add it to both sides:

$$p^2 - 6p + 9 = -7 + 9$$

$$(p - 3)^2 = 2$$

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SOLVE: $3p - 5 = (p - 1)(p - 2)$

Now let's add it to both sides:

$$p^2 - 6p + 9 = -7 + 9$$

$$(p - 3)^2 = 2$$

3. Take the square roots of both sides:

$$(p - 3) = \pm\sqrt{2}$$

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SOLVE: $3p - 5 = (p - 1)(p - 2)$

Now let's add it to both sides:

$$p^2 - 6p + 9 = -7 + 9$$

$$(p - 3)^2 = 2$$

3. Take the square roots of both sides:

$$(p - 3) = \pm\sqrt{2}$$

4. Solve for *r*.

$$p = 3 \pm \sqrt{2}$$

Solution: $p = 3 + \sqrt{2}$ or $p = 3 - \sqrt{2}$

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CLASS ACTIVITY: REALTIMEBOARD

- Solve the quadratic equations by completing the square.

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CLASS ACTIVITY: YOTEACH!

- Q1: When is it better to use completing the square over factorization?

- Q2: Share an example of when both completing the square and factorization would be difficult.

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QUADRATIC FORMULA

Sometimes, it may just be easier to plug in values into a formula and calculate the solution.

The quadratic formula allows us to do that!

To derive it, let's start by completing the square for the generic quadratic equation: $ax^2 + bx + c = 0$

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QUADRATIC FORMULA DERIVATION

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

QUADRATIC FORMULA DERIVATION

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

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QUADRATIC FORMULA DERIVATION

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

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QUADRATIC FORMULA DERIVATION

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

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QUADRATIC FORMULA DERIVATION

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boldsymbol{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

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QUADRATIC FORMULA

Therefore, for any quadratic equation $ax^2 + bx + c = 0$, the roots of given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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SOLVE: $x^2 + 7x + 6 = 0$

Firstly, let's identify *a*, *b* and *c*.

$a = 1$ $b = 7$ $c = 6$

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SOLVE: $x^2 + 7x + 6 = 0$

Firstly, let's identify *a*, *b* and *c*.

$$a = 1 \qquad b = 7 \qquad c = 6$$

Now, let's plug this into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{49 - 24}}{2} = \frac{-7 \pm \sqrt{25}}{2}$$

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SOLVE: $x^2 + 7x + 6 = 0$

$$x = \frac{-7 \pm \sqrt{25}}{2}$$

$$= \frac{-7 \pm 5}{2}$$

SOLVE: $x^2 + 7x + 6 = 0$

$$x = \frac{-7 \pm \sqrt{25}}{2}$$

$$= \frac{-7 \pm 5}{2}$$

$$x = \frac{-7 + 5}{2} \quad \text{or} \quad x = \frac{-7 - 5}{2}$$

$$x = -1 \qquad x = -6$$

Solution: $x = -1$ or $x = -6$

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SOLVE: $2m^2 + m - 10 = 0$

Firstly, let's identify *a*, *b* and *c*.

$a = 2$ $b = 1$ $c = -10$

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SOLVE: $2m^2 + m - 10 = 0$

Firstly, let's identify *a*, *b* and *c*.

$$a = 2 \qquad b = 1 \qquad c = -10$$

Now, let's plug this into the quadratic formula:

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-10)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1 + 80}}{4} = \frac{-1 \pm \sqrt{81}}{4}$$

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SOLVE: $2m^2 + m - 10 = 0$

$$m = \frac{-1 \pm \sqrt{81}}{4}$$

$$= \frac{-1 \pm 9}{4}$$

SOLVE: $2m^2 + m - 10 = 0$

$$m = \frac{-1 \pm \sqrt{81}}{4}$$

$$= \frac{-1 \pm 9}{4}$$

$$m = \frac{-1 + 9}{4} \quad \text{or} \quad m = \frac{-1 - 9}{4}$$

$$m = 2 \qquad m = -\frac{5}{2}$$

Solution: $m = 2$ or $m = {}^{-5}/_{2}$

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CLASS ACTIVITY: YOTEACH!

- Q1: Share quadratic equations with 0, 1 and 2 roots.

- Q2: How can we tell how many roots a quadratic equation has?

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DISCRIMINANT

- As you have already observed, quadratic equations can have 2 roots, 1 repeated root, or no real roots.

- One way to immediately see how many roots an equation has is through the discriminant.

- We can take this discriminant through the quadratic equation.

DISCRIMINANT

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $\Delta = b^2 - 4ac$

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DISCRIMINANT

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $\Delta = b^2 - 4ac$

If $\Delta = b^2 - 4ac < 0$, there are no real roots.

If $\Delta = b^2 - 4ac = 0$, there is 1 repeated root.

If $\Delta = b^2 - 4ac > 0$, there are 2 roots.

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CLASS ACTIVITY: REALTIMEBOARD

- Solve the quadratic equations using the quadratic formula.

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CLASS ACTIVITY: BADABOOM

- Quadratic Equations Quiz

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CLASS ACTIVITY: YOTEACH!

- Reflection: What have you learnt today?

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LESSON OBJECTIVES

☑ Understand the basic concepts of quadratic equations

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